GROWTH OF VAPOR BUBBLES IN A HIGHLY SUPERHEATED LIQUID

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The rate of growth of a bubble is determined from the self-similar solution of the heat-transfer equation. A comparison is made with the Labuntsov–Scriven approximation.

In order to calculate the transient phase of the decompressive flashing of large masses of a superheated liquid it is necessary to know the laws of vapor-bubble dynamics. In general it is required to solve the hydrodynamical and thermophysical problems simultaneously. Quite a large number of applications can be described within the scope of the thermal approximation [1-3].

In the thermal approximation it is assumed that the process of bubble growth is sustained by heat input spent in vaporization. We neglect inertial and viscous forces. Even these simplifications fail to provide suitable analytical expressions for the bubble radius in a variable pressure field, although this type of problem is completely solvable for numerical calculations. Considering the fact that the thermophysical properties of a superheated liquid have not been adequately studied and, accordingly, the degree of uncertainty of the results of calculations of bubble growth rates is still appreciable, the inappropriateness of a precise machine calculation of the bubble dynamics becomes obvious. It is difficult to obtain approximate analytical estimates within the framework of the conventional approach based on the solution of the transient thermal problem with initial conditions. Also, the existing solutions are unsatisfactory at superheats commensurate with the quantity L/c'. These inadequacies are surmounted in the self-similar solution (SSS) proposed below.

We neglect heat transfer between the vapor and liquid; then the heat-balance equation for a single bubble of radius R takes the form

$$\frac{d}{dt}\left(\frac{4}{3}\pi R^3\rho''\right)=\frac{4\pi R^2}{L}\lambda'\left(\frac{\partial\theta}{\partial x}\right)_0. \tag{1}$$

Here the temperature θ is measured relative to the temperature at infinity. To determine the temperature gradient on the bubble surface $(d\theta/dx)_0 \equiv (d\theta/dx)(x=0)$ in an incompressible liquid it is necessary to solve the equation

$$\left(\frac{1}{a}\frac{\partial}{\partial t}-\frac{\partial^2}{\partial x^2}\right)\theta(x,\ t)=\frac{\partial\theta}{\partial x}\left\{\frac{2}{R+x}+\frac{1}{a}\left[1-\left(\frac{R}{R+x}\right)^2\right]\frac{dR}{dt}+\frac{c'}{L}\left(\frac{\partial\theta}{\partial x}\right)_0\left(\frac{R}{R+x}\right)^2\right\},$$
$$\theta(x=\infty,\ t)=\theta(x>0,\ t=0)=0,\ \theta(x=0,\ t>0)=\theta_0. \tag{2}$$

This equation is written in a Lagrangian coordinate system "affixed" to the bubble surface. The coordinate x is reckoned from the bubble surface. The first term on the right-hand side is the nonlinear part of the spherical Laplacian, the second term is associated with convective flow toward the bubble surface in the moving coordinate system, and the third term refers to the liquid flow in vaporization.

For a constant surface temperature θ_0 and $dR/dt \geq 0$ Eq. (2) has the self-similar solution

$$\theta(x,\ t)=\theta\left(\frac{x}{R(t)}\right)=\theta_0+\left(\frac{\partial\theta}{\partial(x/R)}\right)_0\int\limits_0^{x/R}\exp\left[-\frac{yc'}{(1+y)L}\cdot\left(\frac{\partial\theta}{\partial(x/R)}\right)_0-\frac{y^2(3+y)}{2a(1+y)}R\frac{dR}{dt}\right]\frac{dy}{(1+y)^2}. \tag{3}$$

Making use of the condition $\theta(x=\infty,\ t)=0$, we write the temperature of the bubble surface in terms of the

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dimensionless groups $\tilde{T} = -c'\theta_0/L$, $G = Rc'(\partial\theta/\partial x)_0/L$, $\gamma = d(R^2/2a)/dt \geqslant 0$ in the form

$$\frac{\tilde{T}}{G} = \int_0^1 dy \exp\left[-\frac{\gamma y^2}{(1-y)^2}\left(\frac{3}{2} - y\right) - Gy\right]. \tag{4}$$

We rewrite Eq. (1):

$$\gamma + \frac{R^2}{3a}\frac{d}{dt}\ln\rho'' = G\rho'/\rho''. \tag{5}$$

The absolute value of the surface temperature is taken equal to the saturation temperature and depends on the external pressure. The SSS therefore does not exist in a variable pressure field, but expression (4) is still reasonably accurate in the given situation. In support of this fact we note that Eq. (2) with the term $(d\theta/dt)/a$ discarded yields the expression

$$\frac{\tilde{T}}{G} = \int_0^1 dy \exp\left(-\frac{\gamma y^2}{1-y} - Gy\right), \tag{6}$$

where it is not required that θ_0 be constant.

It is seen at once that expressions (4) and (6) coincide for $\gamma \ll 1$ as well as for $G \gg 1$. In the intermediate range of values of the parameters the quantity G calculated according to (4) and (6) does not differ by more than 50%. Consequently, the processes associated with redistribution of the temperature field with the variation of θ_0 are usually negligible in the approximate calculation G.

The system of equations (4) and (5) completely determines the adopted bubble dynamics model. Here the dimensionless values of the temperature gradient G and the rate of change of the bubble surface area are unknown. The density of the vapor ρ'', the density of the liquid ρ', and the relative heat stored by the liquid $\tilde{T}$ are considered to be given functions of the pressure. The differential temperature is equal to the saturated vapor temperature relative to the temperature at infinity, at the instantaneous pressure. An analogous model for a constant vapor density has been investigated in [1], but the approximation used there for the heat balance of the bubble is not valid in the case of high superheats of the liquid.

The integral (4) has been analyzed in [1], in which the authors have obtained a formal series expansion in powers of $\sqrt{\gamma}$ and use explicitly only the first term of the expansion, also writing its asymptotic representation up to $1/\gamma$. These results can be improved. If we replace the first term in the argument of the exponential by $3\gamma y^2/2$, we obtain a fairly strong and useful upper bound on the value of $\tilde{T}/G$:

$$\sqrt{\frac{\pi}{6\gamma}}\exp\left(\frac{G^2}{6\gamma}\right)\left[\Phi\left(\frac{G}{\sqrt{6\gamma}} + \sqrt{\frac{3\gamma}{2}}\right) - \Phi\left(\frac{G}{\sqrt{6\gamma}}\right)\right] \geqslant \frac{\tilde{T}}{G}. \tag{7}$$

Here $\Phi(x)$ is the probability integral [4]. The upper bound represents the exact value of the integral (4) for $\gamma \gg 1$ or for $\gamma \ll 1$. For Eq. (4) we can obtain the asymptotic representation

$$\frac{1}{1-\tilde{T}} \approx \frac{G^2}{3\gamma}\left[1 + \frac{6}{G}\left(1 - \frac{G}{3\gamma}\right) + O(G^{-2}) + O(\gamma^{-2}) + O\left(\frac{1}{\gamma G}\right)\right], \tag{8}$$

$$\tilde{T} \approx G\left[1 - \sqrt{\frac{\pi}{2}\gamma} + O(\gamma)\right]. \tag{9}$$

Moreover, for $3\gamma/2 \gg 1$ we obtain from inequality (7)

$$\tilde{T} \lesssim G\sqrt{\frac{\pi}{6\gamma}}\left[1 - \frac{2G}{\sqrt{\pi 6\gamma}} + O\left(\frac{G^2}{6\gamma}\right)\right]. \tag{10}$$

In expressions (8)-(10) the symbol $O(x^\nu)$ signifies that terms of order x^ν and higher are discarded.

We now consider the case of bubble growth for a constant external pressure, so that Eq. (5) is simplified:

$$G = \rho''\gamma/\rho'. \tag{11}$$

Substituting (11) into (4) or into (7)-(10), we obtain equations in γ, the solutions of which yield the rate of growth of the bubble surface. In particular,

$$\gamma(1-\tilde{T}\ll 1)\approx\frac{3}{1-\tilde{T}}\left(\frac{\rho'}{\rho''}\right)^2\left[1+2\frac{\rho''}{\rho'}(1-\tilde{T})\left(\frac{\rho''}{3\rho'}-1\right)+O((1-\tilde{T})^2)\right], \tag{12}$$

$$\gamma\left(\frac{\rho'\tilde{T}}{\rho''}\ll 1\right)\approx\tilde{T}\frac{\rho'}{\rho''}\left[1+\sqrt{\frac{\pi\rho'}{2\rho''}}\tilde{T}+O(\rho'\tilde{T}/\rho'')\right], \tag{13}$$

$$\gamma\left(\tilde{T}\ll 1,\ \frac{\rho'}{\rho''}\tilde{T}\gg 1\right)\gtrsim\frac{6}{\pi}\left[\frac{\rho'\tilde{T}}{\rho''\left(1-\frac{2}{\pi}\tilde{T}\right)}\right]^2\left[1+O(\tilde{T}^2)+O\left(\frac{\rho''}{\rho'\tilde{T}}\right)\right]. \tag{14}$$

We compare these results with the well-known Labuntsov–Scriven approximation [2]:

$$\gamma_L=\frac{6}{\pi}\left(\frac{\rho'\tilde{T}}{\rho''}\right)^2\left[1+\frac{1}{2}\left(\frac{\pi\rho''}{6\rho'\tilde{T}}\right)^{2/3}+\frac{\pi\rho''}{6\rho'\tilde{T}}\right]. \tag{15}$$

It coincides with the solution in [1] for large and small Jakob numbers $\rho'\tilde{T}/\rho''$. The approximation (15) has the drawback that it does not diverge as $\tilde{T}\to 1$.

It is readily perceived that (13) coincides with the corresponding asymptotic version (15) up to the second term in the brackets of (13). Expression (14) coincides with (15) for $\rho'\tilde{T}/\rho''\gg 1$ only if in the latter we make the substitution

$$\tilde{T}\to\tilde{T}/(1-2\tilde{T}/\pi). \tag{16}$$

In the approximation (15) there is no analog to the asymptotic representation (12). The divergence in (12) as $\tilde{T}\to 1$ is induced by the fact that the heat stored by the liquid, $\theta_0 c'$, is sufficient for its complete vaporization. What this means in practical terms is that the Rayleigh phase of the bubble-growth process lasts indefinitely. For high superheats of the liquid the approximation (15) decreases substantially the bubble growth rate. Numerical calculations have shown that in the range of parameters $2\tilde{T}\to\sqrt{\rho''/\rho'}$ even the upper bound (7) is more precise than (15).

The approximation (15) can be adjusted for high superheats by replacing the heat of vaporization L with an effective value. For $1-\tilde{T}\ll 1$ and $\rho''/\rho'\ll 1$ we have

$$L_{ef}=L\sqrt{\frac{2}{\pi}(1-\tilde{T})}. \tag{17}$$

At the attainable superheat for a number of organic liquids at atmospheric pressure the dimensionless group $\tilde{T}\gtrsim 1$. We note that calculations according to the value of the specific heat along the saturation line give too low a value for this quantity, because the specific heat of the liquid increases with the superheat. For practical calculations we can use the specific heat averaged over the temperature along the saturation line. The replacement of L by L_{ef} is a consequence of including the third term on the right-hand side of Eq. (2). For a broader range of values of ρ''/ρ' and $\tilde{T}$ it is more practical to replace L in the approximation (15) by

$$L_{ef}=(1-\tilde{T})L. \tag{18}$$

This substitution, however, gives an incorrect asymptotic behavior as $\tilde{T}\to 1$.

We consider the growth of a bubble for a variable external pressure. We assume that the pressure in the superheated liquid increases with time in such a way as to maintain a constant bubble radius, i.e., $\gamma=0$, so that from (4) and (5) we deduce the relation

$$\frac{R^2}{3a}\frac{d}{dt}\ln\rho''+\frac{\rho'}{\rho''}\ln(1-\tilde{T})=0. \tag{19}$$

We denote by R^* the value of the radius satisfying this equation. Bubbles with $R>R^*$ will be compressed ($\gamma<0$), and those with $R<R^*$ will continue to grow ($\gamma>0$). Consequently, if the pressure on the liquid containing the bubble is modulated in such a way that $(d\rho''/dt)/\ln(1-\tilde{T})=\text{const}<0$, the bubble-size spectrum will narrow toward $R=R^*$.

Regarding the vapor as an ideal gas, from (19) we obtain

$$R^* = \sqrt{\frac{3a\rho'}{\rho''}\tau_p \ln(1-\tilde{T})^{-1}}, \tag{20}$$

where $\tau_p = \rho''/(d\rho''/dt)$ is the characteristic time scale of the pressure rise. For water superheated to the temperature of intense fluctuation nucleation at atmospheric pressure in processes with $\tau_p = 10^{-4}$ sec we have $R^* \approx 0.2$ mm.

For large bubbles $R \gg R^*$ we infer from (5) the mass conservation condition $R^3(t)\rho(p(t)) = \text{const}$. If $R \ll R^*$, then Eq. (11) remains valid. Using any of the asymptotic expressions (12)-(14) or approximation (15) with L replaced by L_{ef} therein, we can write $R^2(t)$ as a functional of $\tilde{T}$ and ρ''/ρ'. In particular, for $1 - \tilde{T} \ll 1$ we have

$$R^2(t) \approx 6a \int_0^t \frac{(\rho'/\rho'')^2}{1-\tilde{T}} d\tau. \tag{21}$$

Here the quantities in the integrand depend on the pressure, which in turn is a function of the running time τ. The functional (21) is suitable for calculations in the case of an arbitrary but slow variation of the pressure, in which case the second term in (5) can be neglected.

The selected approximation affords the theoretical possibility of calculating analytically the explosive flashing of large masses of liquids, when there is a mutual influence between the pressure field and the vaporization kinetics. It is found in this case that with an increase in the superheat the Rayleigh phase of the growth of a vapor bubble lasts longer than predicted by previous theories.

NOTATION

R, bubble radius; θ, temperature measured relative to the temperature far from the bubble; x, distance from the bubble surface; λ, a, thermal conductivity and thermal diffusivity of the liquid; ρ', ρ'', densities of the liquid and the vapor; c', specific heat of the liquid; L, heat of vaporization; dimensionless groups: $T = -c'\theta(x = 0)/L$, $G = Rc'(\partial\theta/\partial x)/L(x = 0)$, $\gamma = d(R^2/2a)/dt$; p(t), pressure; t, time.

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